

Finding antiderivatives:

a) Suppose $f''(x)=18$, $f'(1)=2$, and $f(1)=3$.

Determine $f(x)$. Since $f''(x)=18$,

$$f'(x) = 18x + C, \quad f'(1) = 2 = 18 + C, \quad C = -16$$

$$f'(x) = 18x - 16, \quad f(x) = 9x^2 - 16x + k,$$

$$f(1) = 3, \quad 3 = 9 - 16 + k, \quad k = 10$$

$$f(x) = 9x^2 - 16x + 10$$

b) Find an antiderivative of $v(x) = \sec^2(6x) + 15x^4$. \leftarrow

$$\frac{d}{dx} \tan x = \sec^2 x, \quad \frac{d}{dx} \tan(6x) = (\sec^2 6x) \cdot 6$$

$$\frac{d}{dx} \frac{\tan(6x)}{6} = \frac{\sec^2(6x) \cdot 6}{6} = \sec^2(6x)$$

$$\frac{d}{dx} (3x^5) = 15x^4,$$

Antiderivative of $v(x)$ is

$$\frac{\tan(6x)}{6} + 3x^5 + C$$

c) Find an antiderivative of $w(x) = \frac{1}{\sqrt{1-4x}}$.

$$\frac{d}{dx} \sqrt{1-4x} = \frac{1}{2\sqrt{1-4x}} \cdot (-4) = \frac{-2}{\sqrt{1-4x}}$$

$$\frac{d}{dx} \frac{\sqrt{1-4x}}{-2} = \frac{-2}{\sqrt{1-4x}} \cdot \frac{1}{-2} = \frac{1}{\sqrt{1-4x}}$$

$-\frac{\sqrt{1-4x}}{2} + C$ is an antiderivative
of $w(x) = \frac{1}{\sqrt{1-4x}}$.